Efficient Co-Domain Quantisation for PDE-Based Image Compression

Laurent Hoeltgen and Michael Breuß
Chair for Applied Mathematics, BTU Cottbus-Senftenberg

Brandenburg
University of Technology
Cottbus - Senftenberg

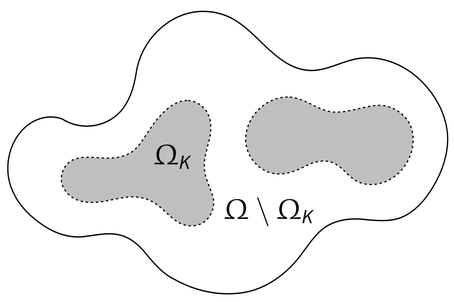
PDE-Based Image Compression

- Beats JPEG and JPEG 2000 for many cases
- Uses point cloud of pixels with corresponding colour data
- Position and value of colours must be optimised
- Optimal colours should minimise error of reconstruction:
  1. Least squares models yield colour values in $\mathbb{R}$
  2. Probabilistic searching strategies return colours as subset of $\mathbb{N}$
- Saving colours in full precision is too memory-expensive
- Quantisation strategies are mandatory, but:
  Slow: run time of hours or more
  Inflexible: impossible to optimise number *and* distribution of colours

Recover image by solving

$$\begin{cases} -\Delta u = 0, & \text{on } \Omega \setminus \Omega_K \\ u = f, & \text{on } \partial\Omega_K \\ \partial_n u = 0, & \text{on } \partial\Omega \setminus \partial\Omega_K \end{cases}$$

with Ω image domain and Ω_K known data



Our Contributions

- Investigate clustering strategies as new approach to quantise the data for compression purposes
- Provide a fast alternative to well established methods

Which Features Do We Consider?

1. Colour information at mask positions
2. Colour histogram of mask positions
3. Colour data of all pixels
4. Full image histogram
5. Colour and positional data of mask pixels
6. Colour and positional data of all pixels

Which Methods Do We Try?

1. K-means (partitional clustering)
2. Successive merging based on Ward's criteria (hierarchical clustering)
3. Probabilistic (fuzzy) clustering with Gaussian mixture models

How Do We Evaluate the Results?

1. Number of clusters
2. Mean squared error of the reconstruction
3. Silhouette plots (adequate measure in clustering contexts)
4. Stability and reliability
5. Run time

What Test Data Do We Use?

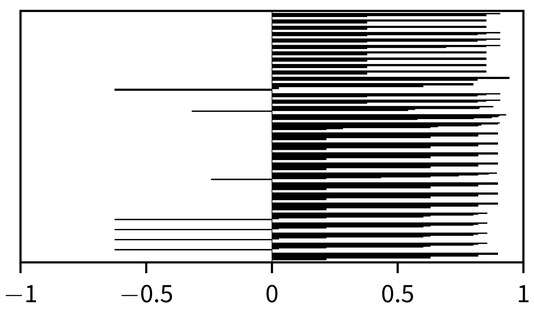
trui test image with optimised mask (5% density)

Findings

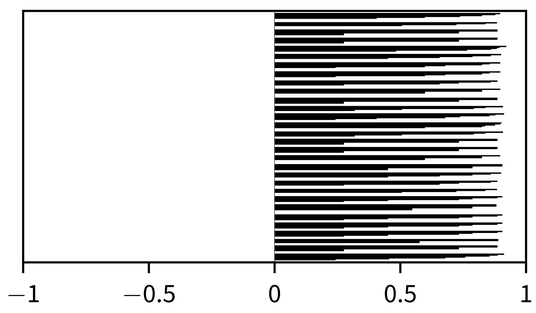
- All methods performed best when applied to colour information at mask positions only
- Mean squared error for best performing methods:

K-means (30 colours) outperforms standard reconstruction (180 colours)!

- Silhouette Plots for 30 clusters (left: hierarchical, right: k-means):
  - K-means clustering better than hierarchical clustering
  - Silhouette plots correlate with mean squared error



Hierarchical clustering with 30 clusters
Mean = 0.59, Median = 0.8



K-means clustering with 30 clusters
Mean = 0.63, Median = 0.73

- K-means yields best results
- Hierarchical Clustering is quite stable
- Gaussian Mixture Models are too unreliable
- Run times range from 0.15 to 0.35 seconds

K-means reconstruction
with 30 colours

Conclusions

- Reduction of number of colours by 80% without loss of accuracy
- Higher compression rates possible due to less colours to store
- Significantly faster PDE-based codecs are possible

References

[1] C. Schmaltz et al.: *Beating the Quality of JPEG 2000 with Anisotropic Diffusion*, 2009
[2] L. Hoeltgen: *Optimal Interpolation Data for Image Reconstructions*, 2015

Future Work

- Check more features and approaches
- Develop working image compression codec (in progress)